

Softly Broken Supersymmetric Gauge-Higgs-Yukawa Theories as Renormalizable Composite Models

Tatsuo Kobayashi^{a 1} and Haruhiko Terao^{b 2}

^aDepartment of Physics, Kyoto University
Kyoto 606-8502, Japan

^bInstitute for Theoretical Physics, Kanazawa University
Kanazawa 920-1192, Japan

Abstract

We examine the (softly broken) supersymmetric gauge-Higgs-Yukawa theories satisfying the compositeness conditions at a certain scale. In these theories the Higgs superfields can be regarded as the chiral composite fields. It is found that there are the fundamental theories, which contain the dimension 5 interactions and the hard SUSY breaking in perturbation, turn to be renormalizable and also softly broken theories in the nonperturbative framework. The soft SUSY breaking parameters as well as the Yukawa coupling in the corresponding gauge-Higgs-Yukawa theories are restricted by the renormalization group invariant relations.

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¹E-mail: kobayashi@gauge.scphys.kyoto-u.ac.jp

²E-mail: terao@hep.s.kanazawa-u.ac.jp

1 Introduction

Supersymmetric extension of the Standard Model (SM) brings much more parameters after supersymmetry (SUSY) breaking than in the SM. Most of them are soft SUSY breaking parameters. It is clearly attractive to reduce these plenty of free parameters by some model independent mechanisms. Especially it is an important problem to clarify the SUSY breaking origins leading the viable phenomenological models. So far scenarios based on the gravity-mediated and the gauge-mediated supersymmetry breaking mechanisms have been mostly investigated. Recently, however, there is a great interest in the anomaly-mediated supersymmetry breaking [1], which restricts the soft SUSY breaking parameters to the renormalization group (RG) invariant relations [2, 3, 4, 5]. Therefore this SUSY breaking mechanism leads to highly predictable phenomenological models. However there is the problem that the slepton masses become tachyonic. Therefore, another source of SUSY breaking has to be introduced in order to resolve this problem [1]³.

In this paper we propose another possibility to reduce the number of the free parameters in supersymmetric theories. We consider the supersymmetric gauge-Higgs-Yukawa theories, whose Higgs chiral superfields are composites of the fundamental matter chiral superfields. The scenarios beyond the SM with composite Higgs particles, e.g. Technicolor models, Top condensation models and so on, have been intensively examined before. In the context of supersymmetry also, the dynamics of composite particles in the Nambu-Jona-Lasinio (NJL) models has been studied [7, 8]. The Higgs particles are given as composites of fermions in these models. The models considered in this paper are a kind of supersymmetric extension of the Top condensation models [9]. However the Higgs particles in our models are composites of scalars. In practice we seek for the special solutions of 1-loop RG equations for the (softly broken) gauge-Higgs-Yukawa theories which satisfy the compositeness conditions at a certain scale [9].

The RG equations for the non-supersymmetric gauge-Higgs-Yukawa theories have been examined in the leading order of a modified $1/N_c$ expansion in Ref. [10]. It was found that the nontrivial continuum limits exist for a certain class of theories. Moreover the gauged NJL models become equivalent to the nontrivial gauge-Higgs-Yukawa theories in the infinite cutoff limit. This proves the renormalizability of the gauged NJL models in four dimensions. Later this renormalizability was also confirmed directly by applying the Exact RG to the gauged NJL models [11]. The Yukawa coupling of the theory equivalent to the renormalizable gauged NJL model is restricted to the Pendelton-Ross fixed point [12]. The fixed point solution has been also obtained by the coupling reduction [13, 14]. The reduction of couplings is a natural consequence, since the gauged NJL models contain less parameters than the gauge-Higgs-Yukawa theories.

If we consider the supersymmetric extension of the gauge-Higgs-Yukawa theories, similar mechanism is found to work as well. In this case the interactions of the fundamental theory are given by dimension 5 operators. It will be shown that these theories also can be renormalizable similarly to the gauged NJL models. Moreover, the SUSY breaking

³ See also Ref.[6], where other contributions are added with respecting the anomaly mediation RG trajectory.

terms are introduced to these theories. Then it is found that a kind of hard SUSY breaking terms can be introduced in the dimension 5 interaction models, while the equivalent gauge-Higgs-Yukawa theories are broken only softly. As interesting results, we will see that not only the Yukawa couplings but also some of the soft SUSY breaking parameters are restricted just by the RG invariant relations. Thus the compositeness requirement can reduce the number of the free parameters in the supersymmetric theories. In the next section we first consider the rigid cases. The SUSY breaking terms are considered in Section 3. Section 4 is devoted to conclusions.

2 Supersymmetric models with composite Higgs

We consider the asymptotically free supersymmetric $SU(N_c)$ gauge theory with $N_f(N_f < 3N_c)$ flavors of "quarks" (Q_i^a, \tilde{Q}_a^i) ($a = 1, \dots, N_c, i = 1, \dots, N_f$) belonging to the fundamental representation. We also introduce the "Higgs" chiral fields H_i^j in (N_f, \bar{N}_f) representation of the flavor $U(N_f) \times U(N_f)$ symmetry. The Lagrangian with Yukawa interactions among quarks and Higgs is given by

$$\begin{aligned} \mathcal{L}_{\text{gHY}} = & \int d^4\theta (Q^{i\dagger} e^V Q_i + \tilde{Q}^i e^V \tilde{Q}_i^\dagger) + \frac{1}{16g^2} \int d^2\theta W^A W_A + \text{h.c.} \\ & + \int d^4\theta H^\dagger H + \int d^2\theta (y \tilde{Q}^j H_j^i Q_i + \frac{m}{2} H_j^i H_i^j) + \text{h.c.} \end{aligned} \quad (1)$$

Here we have assumed that the Yukawa coupling y and the Higgs mass parameter m are flavor independent just for the sake of simplicity. Note that the flavor $U(N_f) \times U(N_f)$ symmetry is broken down into the diagonal one $U(N_f)$ by the last mass term. If the running Yukawa coupling $y(\mu)$ satisfies the so-called compositeness condition at a certain scale Λ (compositeness scale):

$$\lim_{\mu \rightarrow \Lambda} \frac{1}{y^2(\mu)} = 0, \quad (2)$$

then the Higgs fields are reduced to be composite auxiliary fields $H_i^j \sim Q_i \tilde{Q}^j$ at Λ . After integrating out these fields we obtain the gauge theory with dimension 5 interactions:

$$\begin{aligned} \mathcal{L}_{\text{D5}} = & \int d^4\theta (Q^{i\dagger} e^V Q_i + \tilde{Q}^i e^V \tilde{Q}_i^\dagger) + \frac{1}{16g^2} \int d^2\theta W^A W_A + \text{h.c.} \\ & - \int d^2\theta \frac{h}{2} (Q_j \tilde{Q}^i)(Q_i \tilde{Q}^j) + \text{h.c.}, \end{aligned} \quad (3)$$

where the coupling h is given by

$$\lim_{\mu \rightarrow \Lambda} \frac{y^2(\mu)}{m(\mu)} = h. \quad (4)$$

In the followings we examine the 1-loop RG equations for the gauge-Higgs-Yukawa theory given by Eq. (1) and the solutions satisfying the compositeness condition (2).

The beta functions for the N=1 SUSY gauge theories have been known to be given exactly in terms of the anomalous dimensions of the chiral matter fields. For our example they are found to be

$$\begin{aligned}\beta_g &= \mu \frac{dg}{d\mu} = -\frac{g^3}{16\pi^2} \frac{3N_c - N_f + N_f \gamma_Q}{1 - (N_c/8\pi^2)g^2} \\ \beta_y &= \mu \frac{dy}{d\mu} = (2\gamma_Q + \gamma_H)y \\ \beta_m &= \mu \frac{dm}{d\mu} = 2\gamma_H m,\end{aligned}\tag{5}$$

where γ_Q ($= \gamma_{\tilde{Q}}$) and γ_H are the anomalous dimensions of the chiral fields Q (\tilde{Q}) and H respectively. The wave function renormalization factors are given by

$$\gamma_Q = \frac{1}{2} \frac{d \ln Z_Q}{d \ln \mu}, \quad \gamma_H = \frac{1}{2} \frac{d \ln Z_H}{d \ln \mu}.\tag{6}$$

In the 1-loop perturbation the anomalous dimensions are given explicitly as follows:

$$\gamma_Q = \frac{1}{16\pi^2} \left(-\frac{N_c^2 - 1}{N_c} g^2 + N_f y \bar{y} \right), \quad \gamma_H = \frac{N_c}{16\pi^2} y \bar{y}.\tag{7}$$

Therefore we obtain the 1-loop beta functions for $\alpha = g^2/8\pi^2$, $\alpha_y = y\bar{y}/8\pi^2$ and m as

$$\frac{d\alpha}{dt} = -b\alpha^2,\tag{8}$$

$$\frac{d\alpha_y}{dt} = (a\alpha_y - c\alpha)\alpha_y,\tag{9}$$

$$\frac{dm}{dt} = d\alpha_y m,\tag{10}$$

where we have introduced new parameters; $t = \ln(\mu/\mu_0)$, $a = 2N_f + N_c$, $b = 3N_c - N_f$, $c = 2(N_c^2 - 1)/N_c$ and $d = N_c$.

The general solutions of Eq. (8) and (9) have been analyzed in Ref. [12, 10]. The solutions of Eq. (8) are given by

$$\alpha(t) = \frac{\alpha(0)}{1 + b\alpha(0)t}.\tag{11}$$

By noting that the following quantity

$$\Gamma = \alpha^{\frac{c-b}{b}} \left(1 - \frac{c-b}{a} \frac{\alpha}{\alpha_y} \right)\tag{12}$$

gives a RG invariant, the general solutions of Eq. (9) are easily found to be

$$\alpha_y(t) = \frac{c-b}{a} \alpha(t) \left(1 + h_0 \alpha^{-\frac{c-b}{b}}(t) \right)^{-1},\tag{13}$$

where h_0 is an integration constant. The special solution with $h_0 = 0$, $\alpha_y^*(t) = (c-b)/b\alpha(t)$, is known as the Pendleton-Ross "fixed point" [12]. In the case of $c > b$, the Yukawa interaction is nontrivial if and only if $h_0 \geq 0$. Therefore if the theory is nontrivial, then the Yukawa coupling at the IR scale μ_0 should be observed in the region [14, 10]

$$0 < \alpha_y(0) \leq \alpha_y^*(0) = \frac{c-b}{a}\alpha(0). \quad (14)$$

While, for $-\alpha^{(c-b)/b}(0) < h_0 < 0$ ⁴, there exists a Landau pole at a finite value of $t > 0$. Thus the solution satisfying the compositeness condition (2) at $t = t_\Lambda = \ln(\Lambda/\mu_0)$ is found to be

$$\alpha_y(t; t_\Lambda) = \frac{c-b}{a}\alpha(t) \left(1 - \left(\frac{\alpha(t)}{\alpha(t_\Lambda)} \right)^{-\frac{c-b}{b}} \right)^{-1}. \quad (15)$$

The Yukawa coupling of the gauge-Higgs-Yukawa theory equivalent to the dimension 5 interaction model given at the scale Λ is fixed to this value. It should be noted that the sequence of theories parameterized by Λ converges to a nontrivial theory in the $\Lambda \rightarrow \infty$ limit. This implies the (nonperturbative) renormalizability of the dimension 5 interaction models, since the models possess nontrivial continuum limits, though perturbatively nonrenormalizable. The Yukawa coupling corresponding to these renormalizable models is given by the "fixed point" value $\alpha_y^*(t)$. Such a nontrivial limit is realized only when $c > b$, ($N_c + 2/N_c < N_f < 3N_c$). Hereafter we restrict ourselves to this case.

Here let us sketch how the quantum corrections are treated in order to make the dimension 5 interaction models renormalizable. The nonrenormalization theorem for supersymmetric theories holds even in the presence of nonrenormalizable couplings [15]. Therefore the superpotential is protected from quantum corrections and we need to consider only the Kähler potential part of Q and \bar{Q} . In the 2-loop calculation the dimension 5 interaction causes the quadratic divergence in the wave function renormalization factor Z_Q [16]. Similarly higher powers of divergences are generated through the higher loop corrections. This is the reason of the nonrenormalizability in the perturbative calculations. However if we sum up the chain diagrams shown in Fig. 1 first, then the four point function is given by the tree diagram of the composite Higgs exchange⁵. As a result, the power of divergence in the corrections for the two point function $\langle QQ^\dagger \rangle$ as shown in Fig. 2 is seen to be tamed down to logarithmic one. Of course we do not say that the renormalizability proof is given by this naive argument. However the nonperturbative calculations have been explicitly performed for the gauged NJL models by using the Exact RG, which is a sort of the Wilson RG [11]. In practice it has been found that the renormalized trajectory exists as long as $c > b$. This implies the nonperturbative renormalizability of the models. Here, however, we are not going to address to such a nonperturbative analysis for the supersymmetric theories.

In practice the nontriviality of the Yukawa coupling is not sufficient for the proof of renormalizability. It is necessary to show that the supersymmetric mass parameter m

⁴We should exclude $h_0 < -\alpha^{(c-b)/b}(0)$ so as for $\alpha_y(0)$ to be finite and positive.

⁵In the leading order of large N_c expansion, the four point function is evaluated in this way.

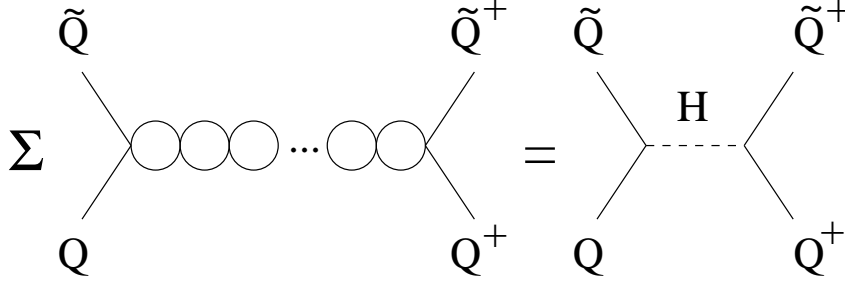


Figure 1: The nonperturbative sum of the chain diagrams for the four point function.

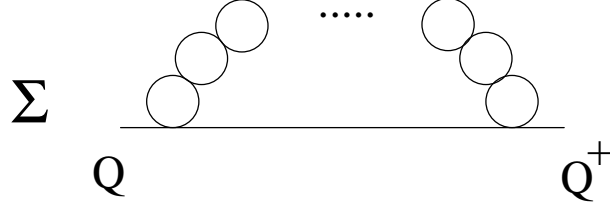


Figure 2: The nonperturbative corrections to the two point function.

also can be kept finite in the limit of $\Lambda \rightarrow \infty$. If we can make the mass finite by tuning the coupling h properly, then it will be shown that the dimension 5 interaction model is equivalent to the nontrivial gauge-Higgs-Yukawa theory, therefore is renormalizable. In order to see this, we first consider the renormalization of the mass parameter m in the gauge-Higgs-Yukawa theory. The general solution of Eq. (10) is also found from the RG invariant

$$\Omega = m \left(\alpha_y \alpha^{-\frac{c}{b}} \right)^{-\frac{d}{a}}. \quad (16)$$

Therefore the mass parameter at scale t is given by

$$m(t) = m(0) \left(\frac{\alpha(t)}{\alpha(0)} \right)^{-\frac{cd}{ab}} \left(\frac{\alpha_y(t)}{\alpha_y(0)} \right)^{\frac{d}{a}}. \quad (17)$$

It is seen that $m(t)$ also diverges as $t \rightarrow t_\Lambda$.

The parameter of the dimension 5 interaction at the compositeness scale is given by

$$|h|^2 = \frac{1}{\Lambda^2} |\hat{h}(t_\Lambda)|^2 = (8\pi^2)^2 \lim_{t \rightarrow t_\Lambda} \frac{\alpha_y^2(t)}{|m(t)|^2}, \quad (18)$$

where we have introduced the dimensionless parameter \hat{h} . Therefore the mass parameter is related with the bare coupling $\hat{h}(t_\Lambda)$ by

$$|m(t)|^2 = (8\pi^2)^2 \alpha_y^2(t) \left(\frac{\alpha_y(t_\Lambda)}{\alpha_y(t)} \right)^{2-\frac{2d}{a}} \left(\frac{\alpha(t_\Lambda)}{\alpha(t)} \right)^{\frac{2cd}{ab}} \frac{\Lambda^2}{|\hat{h}(t_\Lambda)|^2}. \quad (19)$$

Indeed this expression is rather formal, since $\alpha_y(t_\Lambda)$ is diverging. However we can tune the bare coupling $\hat{h}(t_\Lambda)$ also diverging so that the mass parameter $m(t)$ is maintained to be finite. This property is a contrast with the four fermi coupling in the gauged NJL model, which is fine tuned to a finite critical value [10, 11]. Also the Λ dependence of $m(t)$ is removed by adjusting the bare coupling $\hat{h}(t_\Lambda)$ properly. The renormalization of $\hat{h}(t_\Lambda)$ is performed as

$$\frac{\mu}{\hat{h}(t)} = \left(\frac{\alpha_y(t_\Lambda)}{\alpha_y(t)} \right)^{1-\frac{d}{a}} \left(\frac{\alpha(t_\Lambda)}{\alpha(t)} \right)^{\frac{cd}{ab}} \frac{\Lambda}{\hat{h}(t_\Lambda)}. \quad (20)$$

With this renormalized coupling the mass parameter is expressed by

$$|m(t)|^2 = (8\pi^2)^2 \mu^2 \frac{\alpha_y^2(t)}{|\hat{h}(t)|^2}. \quad (21)$$

Thus we have shown that the dimension 5 interaction models are nonperturbatively renormalizable and the equivalent gauge-Higgs-Yukawa theories must satisfy the IR fixed point relation: $\alpha_y(t)/\alpha(t) = (c - b)/b$. Such reduction of couplings [13, 14] seems to necessarily happen, since the dimension 5 interaction model contains less number of the free parameters than the gauge-Higgs-Yukawa theory does.

In the rest of this section let us make a brief comment on the relation to the magnetic description of SQCD in the conformal window, $(3/2)N_c < N_f < 3N_c$ [17]. It has been known that the gauge-Higgs-Yukawa theories given by Eq. (1) possess a nontrivial IR fixed point and the IR dynamics is equivalently described by the $SU(N_f - N_c)$ SQCD with N_f flavors, if $m = 0$. The IR fixed point appears in the weak coupling region for $3N_c = (1 + \epsilon)N_f$ ($\epsilon \ll 1$) and therefore perturbation is reliable. By using the anomalous dimensions evaluated at the 1-loop order given in Eq. (7), we find the fixed points at

$$\begin{aligned} N_c(\alpha^*, \alpha_y^*) &= (0, 0) && \text{UV fixed point,} \\ &= (0, \epsilon) && \text{Unstable fixed point,} \\ &= (2\epsilon, 7\epsilon) && \text{IR fixed point,} \end{aligned} \quad (22)$$

where we have assumed $N_c, N_f \gg 1$ [18]⁶. Then the straight line connecting the UV fixed point and the IR fixed point is just the special solution satisfying $\alpha_y(t)/\alpha(t) = (c - b)/a$ [19]. Therefore the RG analysis performed in this section implies that the IR dynamics of the dimension 5 interaction model is described by the theory on the IR fixed point added the mass perturbation, $\int d\theta^2 HH + \text{h.c.}$. On the other hand the fixed point theory has the dual description by the $SU(N_f - N_c)$ gauge theory. It is, therefore, speculated that the IR dynamics of the dimension 5 interaction model is also described by the dual gauge theory perturbed with the corresponding interaction $\int d\theta^2 (q\tilde{q})^2 + \text{h.c.}$, where (q, \tilde{q}) are the dual quarks of $SU(N_f - N_c)$ gauge theory. This operator is found to have the same dimension as $\int d\theta^2 HH$ and, therefore, is relevant at the IR fixed point.

⁶The RG flow diagram is described in Ref. [18].

As N_f reduces, the nontrivial fixed points moves towards the strongly coupled region. In such cases perturbative calculation is not applicable. However we can deduce some results from the exact RG equations given by Eq. (5). First it is seen that the anomalous dimensions at the IR fixed point are exactly obtained as

$$\gamma_Q^* = -\frac{1}{2}\gamma_H^* = -\frac{3N_c - N_f}{N_f}, \quad (23)$$

and that the Yukawa coupling becomes exactly marginal there. Therefore the mass parameter m , whose dimension is given by $1 - \gamma_H$, is relevant for $N_f > 2N_c$. However it turns irrelevant for $N_f < 2N_c$ and the composite Higgs particles H become massless in the IR. On the other hand the coupling of the dimension 5 interaction h carries the dimension of $-1 - 2\gamma_Q$. Hence h appears as a relevant coupling for $N_f < 2N_c$, while it is irrelevant for $N_f > 2N_c$. Taking into account that the flavor symmetry $U(N_f) \times U(N_f)$ of the gauge-Higgs-Yukawa theory is broken to diagonal $U(N_f)$ by the mass term, we should suppose that the operator $\int d^2\theta (Q\tilde{Q})^2$ is generated in the IR by nonperturbative dynamics, since it is the relevant operator allowed by the symmetry. Then the IR fixed point plays a role of the UV fixed point for the relevant perturbation by h . In this case the renormalizability of the theory with the "nonrenormalizable" interaction $\int d^2\theta (Q\tilde{Q})^2$ is guaranteed by the presence of the nontrivial UV fixed point.

3 Soft SUSY breaking

The spurion technique is very useful to incorporate the soft SUSY breaking parameters [20]. Also it has been known that the beta functions for the soft breaking parameters can be obtained immediately from the beta functions for the rigid theories [21, 3, 22, 23]. For the gauge-Higgs-Yukawa theory examined in the previous section we can introduce five soft SUSY breaking parameters $(m_Q^2, m_H^2, m_g, A, B)$. The Lagrangian is given as

$$\begin{aligned} \mathcal{L}_{\text{gHY}} = & \int d^4\theta (1 - m_Q^2 \eta \bar{\eta}) (Q^{i\dagger} e^V Q_i + \tilde{Q}^i e^V \tilde{Q}_i^\dagger) + \int d^4\theta (1 - m_H^2 \eta \bar{\eta}) H^\dagger H \\ & + \frac{1}{16g^2} \int d^2\theta (1 - 2m_g \eta) W^A W_A + \text{h.c.} \\ & + \int d^2\theta \left(y(1 - A\eta) H_j^i Q_i \tilde{Q}^j + \frac{m}{2} (1 - B\eta) H_j^i H_i^j \right) + \text{h.c.}, \end{aligned} \quad (24)$$

by using the spurion superfields $\eta = \theta^2, \bar{\eta} = \bar{\theta}^2$. We also define the chiral superfields

$$\begin{aligned} S &= \frac{1}{g^2} (1 - 2m_g \eta), \\ Y &= y(1 - A\eta), \\ M &= m(1 - B\eta), \end{aligned} \quad (25)$$

for the later conveniences. On the other hand we suppose to introduce the supersymmetry breaking parameters in the dimension 5 interaction model as

$$\mathcal{L}_{\text{D5}} = \int d^4\theta (1 - m_Q^2 \eta \bar{\eta}) (Q^{i\dagger} e^V Q_i + \tilde{Q}^i e^V \tilde{Q}_i^\dagger)$$

$$+\frac{1}{16g^2}\int d^2\theta(1-2m_g\eta)W^AW_A-\int d^2\theta\frac{h}{2}(1-C\eta)(Q_j\tilde{Q}^i)(Q_i\tilde{Q}^j)+\text{h.c.}..(26)$$

In this section we are going to examine whether this model can be also equivalent to the specific case of the softly broken gauge-Higgs-Yukawa theories. Note that the C -term introduced into the Lagrangian (25) gives a SUSY breaking term of four-scalar interaction. In the case of perturbatively renormalizable theories such a term is regarded as hard breaking. However, our findings are as follows. We can show the equivalence between these two types of theories by imposing certain compositeness conditions just as seen in the rigid cases. Therefore it is seen that the dimension 5 interaction models are actually softly broken as well as renormalizable by taking the infinite limit of the compositeness scale. Moreover it is found that the soft SUSY breaking parameters (m_Q^2, m_H^2, A) in the corresponding gauge-Higgs-Yukawa theories are restricted according to the RG invariant relations [3, 4, 5].

The beta functions for the soft SUSY breaking parameters can be obtained by introducing the renormalization superfield $\tilde{Z}_i(i = V, Q, H)$ related to Z_i via the redefinition of coupling constants [21, 23, 3] ⁷

$$\tilde{Z}_i(\alpha, \alpha_y) = Z_i(\tilde{\alpha}, \tilde{\alpha}_y). \quad (27)$$

Here the redefined couplings are given by

$$\tilde{\alpha} = \frac{1}{8\pi^2} [\text{Re}(S)]^{-1} \quad (28)$$

$$= \alpha(1 + m_g\eta + \bar{m}_g\bar{\eta} + 2m_g\bar{m}_g\eta\bar{\eta}),$$

$$\tilde{\alpha}_y = \frac{1}{8\pi^2} Y\bar{Y}(1 + m_Q^2\eta\bar{\eta})^2(1 + m_H^2\eta\bar{\eta}) \quad (29)$$

$$= \alpha_y(1 - A\eta - \bar{A}\bar{\eta} + (A\bar{A} + \Sigma)\eta\bar{\eta}), \quad (30)$$

where $\Sigma = 2m_Q^2 + m_H^2$. By using this fact and the anomalous dimensions given by Eq. (7), the 1-loop beta functions for the couplings $(m_Q^2, m_H^2, m_g, A, B)$ ⁸ are found to be

$$\frac{dm_g}{dt} = -b\alpha m_g, \quad (31)$$

$$\frac{dA}{dt} = a\alpha_y A + c\alpha m_g, \quad (32)$$

$$\frac{dB}{dt} = d\alpha_y A, \quad (33)$$

$$\frac{dm_Q^2}{dt} = -cm_g\bar{m}_g\alpha + N_f(A\bar{A} + \Sigma)\alpha_y, \quad (34)$$

$$\frac{dm_H^2}{dt} = N_c(A\bar{A} + \Sigma)\alpha_y. \quad (35)$$

⁷ The structure of the superfield diagrams restricts the Yukawa coupling dependence of the Z factor so that $\beta_y(\partial Z/\partial y) = \beta_{\bar{y}}(\partial Z/\partial \bar{y})$ [4]. Therefore Z is given in terms of α_y in the rigid case.

⁸The exact beta functions of the soft scalar mass parameters contain an extra regularization scheme dependent term, which is not obtained by this procedure. (See Ref. [3].) However this term does not contribute in the 1-loop order analysis performed here.

Therefore the sum of the soft scalar mass Σ satisfies

$$\frac{d\Sigma}{dt} = -2c\alpha m_g \bar{m}_g + a\alpha_y (A\bar{A} + \Sigma). \quad (36)$$

Now we seek for the general solutions of these RG equations in order to extract the specific RG flows allowed by the compositeness conditions. In the softly broken case also, it is efficient to find out the RG invariants to obtain the general solutions [24]⁹. Interestingly enough the renormalizations of the combinations $\text{Re}(S)$, $\tilde{\alpha}_y$ and $|\tilde{m}|^2 = M\bar{M}(1 + m_H^2 \eta \bar{\eta})^2$ are performed with the renormalization factors given by the redefinition of coupling constants;

$$\text{Re}(S)_{\text{bare}} = Z_V^{-1}(\tilde{\alpha}, \tilde{\alpha}_y) \text{Re}(S)_{\text{ren}}, \quad (37)$$

$$\tilde{\alpha}_{y\text{bare}} = Z_Q^{-2}(\tilde{\alpha}, \tilde{\alpha}_y) Z_H^{-1}(\tilde{\alpha}, \tilde{\alpha}_y) \tilde{\alpha}_{y\text{ren}}, \quad (38)$$

$$|\tilde{m}|_{\text{bare}}^2 = Z_H^{-2}(\tilde{\alpha}, \tilde{\alpha}_y) |\tilde{m}|_{\text{ren}}^2. \quad (39)$$

It should be noted that these are just the same forms of the renormalization in the rigid case and hold in all orders of perturbation. Here let us mention the reason why these renormalizations are satisfied. Eq. (37) follows from the renormalization of the vector superfield V naively. On the other hand the wave function renormalization factors of Q, \tilde{Q} and H must be chiral superfields. Therefore the renormalization superfields $\tilde{Z}_i (i = Q, H)$ given by Eq. (27) should be decomposed into the chiral (antichiral) renormalization superfields $z_i(\bar{z}_i)$ and the renormalization of soft scalar masses Δ_i as

$$\tilde{Z}_i = z_i(1 + \Delta_i \eta \bar{\eta}) \bar{z}_i. \quad (40)$$

The renormalization of the chiral fields Y, M is given in terms of z_i [23] by

$$\begin{aligned} Y_{\text{bare}} &= z_Q^{-2} z_H^{-1} Y_{\text{ren}}, \\ M_{\text{bare}} &= z_H^{-2} M_{\text{ren}}. \end{aligned} \quad (41)$$

The renormalizations for the antichiral superfields are given similarly. The renormalization for the soft scalar masses is also performed by

$$(1 + m_i^2 \eta \bar{\eta})_{\text{bare}} = (1 + \Delta_i \eta \bar{\eta})^{-1} (1 + m_i^2 \eta \bar{\eta})_{\text{ren}}. \quad (42)$$

By taking account of these renormalizations, Eq. (38) and (39) are found to realize.

Therefore the 1-loop beta functions for the above combinations can be immediately written down as

$$\frac{d}{dt} \tilde{\alpha}^{-1} = -b, \quad (43)$$

$$\frac{d}{dt} \tilde{\alpha}_y = (a\tilde{\alpha}_y - c\tilde{\alpha}) \tilde{\alpha}_y, \quad (44)$$

$$\frac{d}{dt} |\tilde{m}|^2 = 2d\tilde{\alpha}_y |\tilde{m}|^2. \quad (45)$$

⁹ See also Ref.[25].

Certainly these equations are found to reproduce Eq. (31-36). The RG invariants for the redefined couplings are also easily obtained, since we have already known the RG invariants in the rigid case as given by Eq. (12) and (16).

First let us consider the RG invariant derived from Eq. (43). It is seen that the η ($\bar{\eta}$) component of the L.H.S. gives a RG invariant m_g/α (\bar{m}_g/α) [2, 3, 4]. Therefore the general solution of the gaugino mass is found to be

$$m_g(t) = m_g(0) \left(\frac{\alpha(t)}{\alpha(0)} \right). \quad (46)$$

Next consider the RG invariant for $\tilde{\alpha}_y$ [24]. The RG invariant Γ given by Eq. (12) is extended to

$$\tilde{\alpha}^{\frac{c-b}{b}} \left(1 - \frac{c-b}{a} \frac{\tilde{\alpha}}{\tilde{\alpha}_y} \right) = \tilde{\Gamma} = -h_0 - h'_0 \eta - \bar{h}'_0 \bar{\eta} + h''_0 \eta \bar{\eta}, \quad (47)$$

which generates a set of RG invariants. These RG invariant quantities present us the general solutions for A and Σ adding to Eq. (13):

$$A(t) = -m_g(t) + \frac{c-b}{b} (R(t) - 1) m_g(t) + h'_0 \alpha^{-\frac{c-b}{b}}(t) R(t), \quad (48)$$

$$\begin{aligned} \Sigma(t) = & |m_g(t)|^2 + (A(t) + m_g(t))(\bar{A}(t) + \bar{m}_g(t)) \\ & + \frac{c-b}{b} \left[(A(t) + m_g(t))\bar{m}_g(t) + (\bar{A}(t) + \bar{m}_g(t))m_g(t) \right] \\ & - \frac{c}{b} \frac{c-b}{b} (R(t) - 1) |m_g(t)|^2 + h''_0 \alpha^{-\frac{c-b}{b}}(t) R(t), \end{aligned} \quad (49)$$

where we have introduced

$$R(t) = \frac{a}{c-b} \frac{\alpha_y(t)}{\alpha(t)}. \quad (50)$$

Similarly we derive a set of RG invariants by extending Eq. (16). The RG invariant

$$|\tilde{m}|^2 \left(\tilde{\alpha}_y \tilde{\alpha}^{-\frac{c}{b}} \right)^{-\frac{2d}{a}} = |\tilde{\Omega}|^2 = \exp(k_0 - k'_0 \eta - \bar{k}'_0 \bar{\eta} + 2k''_0 \eta \bar{\eta}), \quad (51)$$

is found to present us the general solutions for B and m_H^2 as

$$B(t) = \frac{2d}{a} \left(A(t) + \frac{c}{b} m_g(t) \right) + k'_0, \quad (52)$$

$$m_H^2(t) = \frac{d}{a} \left(\Sigma(t) - \frac{c}{b} |m_g(t)|^2 \right) + k''_0. \quad (53)$$

Thus we have obtained the general solutions for all the running coupling constants appearing in the softly broken gauge-Higgs-Yukawa theories.

Now let us consider the compositeness conditions for the soft parameters, i.e.

$$\lim_{\mu \rightarrow \Lambda} \frac{1}{\tilde{\alpha}_y(\mu)} = 0. \quad (54)$$

In order that the gauge-Higgs-Yukawa theory becomes equivalent to the model given by Eq. (25) at the compositeness scale Λ , it is sufficient for all of $A(t)$, $B(t)$, $m_Q^2(t)$, $m_H^2(t)$ to be finite in the limit $t \rightarrow t_\Lambda$. In the expressions of the general solutions $R(t)$ diverges in this limit. Therefore it is enough to fix the integration constants as

$$\begin{aligned} h'_0 &= -\frac{c-b}{b}\alpha^{\frac{c-b}{b}}(t_\Lambda)m_g(t_\Lambda), \\ h''_0 &= \frac{c}{b}\frac{c-b}{b}\alpha^{\frac{c-b}{b}}(t_\Lambda)|m_g(t_\Lambda)|^2. \end{aligned} \quad (55)$$

As a result $A(t)$ and $\Sigma(t)$ are completely fixed, while $B(t)$ and $m_H^2(t)$ remain to be adjustable parameters. This result seems reasonable, since the number of the independent couplings should be equal to that of the dimension 5 interaction model.

If we take the limit $\Lambda \rightarrow \infty$, the dimension 5 interaction models become equivalent to the nontrivial, therefore nonperturbatively renormalizable, and also softly broken gauge-Higgs-Yukawa theories. In these theories the soft parameters A and Σ are reduced to

$$\begin{aligned} A(t) &= -m_g(t), \\ \Sigma(t) &= |m_g(t)|^2, \end{aligned} \quad (56)$$

which are the RG invariant relations [3, 4, 5]. Also the scale dependence of the running couplings $B(t)$ and $m_H^2(t)$ is given by

$$\begin{aligned} B(t) &= \frac{2d}{a}\frac{c-b}{b}m_g(t) + k'_0, \\ m_H^2(t) &= -\frac{d}{a}\frac{c-b}{b}|m_g(t)|^2 + k''_0, \end{aligned} \quad (57)$$

in this limit. It should be noted that these solutions coincide also with the RG invariant relations up to integration constants.

4 Conclusions

In this paper we have studied the special solutions of 1-loop running couplings in the softly broken supersymmetric gauge-Higgs-Yukawa theories satisfying the compositeness conditions. These theories can be regarded as composite Higgs models and the fundamental theories contain dimension 5 interactions. In the limit of infinite composite scale the theories become equivalent to the gauge-Higgs-Yukawa theories with the nontrivial continuum limits. Therefore the dimension 5 interaction models are found to be renormalizable in this nonperturbative sense.

The models contain also a SUSY breaking term of four-scalar interaction, which is regarded as hard breaking if added to the perturbatively renormalizable theories. In the equivalent gauge-Higgs-Yukawa theories, however, supersymmetry is only softly broken. It has been found that the A -parameter and the sum of soft masses Σ as well as the Yukawa coupling of the composite Higgs models are completely fixed by their RG invariant

relations as the composite scale goes to infinity. Thus the number of free parameters can be well reduced by considering the composite models. The solutions for the gaugino mass, the B-parameter and the soft masses contain free parameters adjustable by the bare SUSY breaking parameters in the fundamental theories.

The soft SUSY breaking parameters restricted to the RG invariants are also obtained in the anomaly mediated SUSY breaking scenarios [1]. However in these scenarios the slepton masses are predicted to have negative (mass)², since each of the soft scalar masses is restricted by the RG invariant relation. Therefore, another source of SUSY breaking has to be introduced in order to resolve this problem [1, 6]. In contrast with the anomaly mediated SUSY breaking, the composite Higgs models allow us to adjust each soft scalar mass so that the RG invariant sum rules are unchanged [5]. The soft scalar masses should be determined by the SUSY breaking mechanism in the fundamental models. However the composite Higgs model cannot restrict the couplings other than in the top sector, if applied to the MSSM. Therefore we cannot say anything about the slepton masses in this approach. Rather, we have presented this model as a possible dynamical mechanism to constrain the sum of soft scalar masses as well as the Yukawa and the A-term couplings to the RG invariants ¹⁰.

We have also discussed the IR dynamics of the gauge-Higgs-Yukawa model in relation to the Seiberg duality. In the conformal window, $(3/2)N_c < N_f < 3N_c$, the IR dynamics is controlled by the nontrivial IR fixed point. Specially in the case of $N_f < 2N_c$, the dimension 5 interaction $\int d^2\theta(Q\tilde{Q})^2$ are found to be relevant, though the mass perturbation is irrelevant there. The renormalizability of the theory deformed by this interaction is guaranteed by the nontrivial fixed point.

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¹⁰If even small Yukawa couplings are realized as fixed points, the sum rules are applicable to other sfermions as proposed in Ref.[26].

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